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HEAT TRANSFER OF A SHARP CONE IN A SUPERSONIC RAREFIED GAS FLOW

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Аннотация—Сообщается о методике и результатах опытных исследований равновесной температуры и теплообмена острых конусов (с полууглами при вершине—5°, 10°, 15°, 30°), обтекавшихся разреженным воздухом под нулевым углом атаки при числах Маха $M = 2{,}5$–10; числах Рейнольдса $Re = 25$–5500; числах $(\sqrt{Re})/M(\sqrt{C}) = 1$–$35$.

NOMENCLATURE

- x, distance from the cone top along its generatrix;
- l, length of the cone generatrix;
- t, diameter of the front cone blunt;
- T, T_0, thermodynamic temperature and adiabatic stagnation temperature in undisturbed flow, respectively;
- T_w, cone surface temperature;
- T_{ex}, T_e, local (at point x) and mean (l) equilibrium temperature;
- θ, half-angle at the cone apex;
- u, λ, μ, ρ, velocity, mean free path length, viscosity and gas density in an undisturbed flow, respectively;
- γ, $= c_p/c_v$, isobaric and isochoric heat capacity ratio;
- T_1, thermodynamic gas temperature on the cone surface in non-viscous flow (calculated from tables [13]);
- M, Mach number in undisturbed flow;
- M_1, Mach number on the cone surface in non-viscous flow (determined from tables [13]);
- μ_e, air viscosity at T_e;
- μ_w, air viscosity at T_w;
- C_e, $= (\mu_e/\mu)(T/T_e)$;
- C_w, $= (\mu_w/\mu)(T/T_w)$;
- q_x, local specific flux to the cone;
- α, mean heat-transfer coefficient (equation 3);
- r, $= (T_e - T)/(T_o - T)$, mean temperature-recovery factor;
- r_1, $= (T_e - T_1)/(T_0 - T_1)$, mean temperature-recovery factor calculated with the help of T_1 (based on local M_1);
- r_m, recovery factor in free molecular flow [11];
- r_k, value of r_1 at continual flow without strong interaction between shock wave and boundary layer (equation 4);
- η, $= (\rho u l \theta^4/\mu)^{1/5}$, parameter of interaction [9];
- Kn_t, $= \lambda/t$;
- $\bar{T}_w$, $= T_w/T_0$;
- Re, $u\rho l/\mu$;
- St_x, $= q_x/\rho u c_p (T_{ex} - T_w)$;
- St, $= \alpha/\rho u c_p$;
- Pr, Prandtl number.

INTRODUCTION

THE AIM of the present work was to conduct an experimental study of heat-transfer and equilibrium temperature under the conditions intermediate between those of free-molecular flow around a cone and a regime of Navier–Stokes–Fourier continuum in which a thin laminar boundary layer exists everywhere along the cone element (except the infinitesimal region near the cone apex) and the interaction between the layer and shock wave can be neglected.

The investigation was carried out in a low-density wind tunnel described in references [1, 2]. The working flows were produced by means of conic nozzles. The design and dimensions of four of them (Nos. 1–4) are given elsewhere [2]. The design of the fifth one is similar, the dimensions are as follows: throat dia. is 13 mm; exit section, 33 mm; the total expansion angle of the supersonic portion of the nozzle is 12°. The methods of measuring the working flow parameters are described in detail in papers [1–3].

The working models of cones were made of electrolytic copper. Their designs and dimensions are given in Fig. 1 and Table 1. The model dimensions were selected in accordance with the dimensions of the isentropic core of gas flow in nozzles. Chromel–Copel thermocouples (with wire diameter from 0·1 to 0·15 mm) were attached to the body of each model.

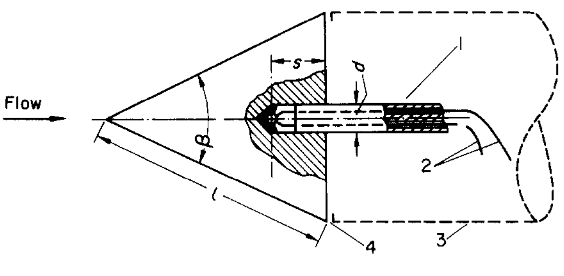

FIG. 1. Details of design of models.
1.—porcelain tube; 2.—thermocouple; 3.—paper (tracing paper) cylinder; 4.—place where cone and cylinder are sticking together.

The thermocouples were calibrated before and after the attachment. After the attachment of the thermocouples, the models were carefully polished. The diameters of all cone blunts measured by microscope were equal to $t \gtrsim 0{\cdot}03$ mm. In tests with nozzles Nos. 1–4 the Knudsen numbers for the front blunt were $Kn_t \approx 10$–25, in tests with nozzle No. 5, $Kn_t \gtrsim 1$. The estimations made on the basis of the continuum theory to define the effect of blunt upon the pattern of the disturbed flow showed that from this point of view the cone models may be considered sharp. The tests carried out with nozzle No. 5 were to a certain degree control ones. According to the estimates the results of these tests should be treated as in the case when continual flow exists practically all over the surface of the cone and there is no strong interaction between the shock wave and boundary layer. The heat-transfer and equilibrium temperature of the cone under these conditions have already been investigated [4–6]. The model was placed in the flow and the angle of attack was controlled by the method described [2]. (All the runs were carried out at zero angle of attack.)

Table 1. Dimensions of cone models (see Fig. 1)

Number of model	β (deg)	l (mm)	d (mm)	s (mm)
1	60	3·19	0·6	1
2	60	4·82	0·6	1
3	60	10·73	1	3
4	60	22·8	2	6
5	30	4·82	0·6	1
6	30	10·15	1	3
7	30	21·1	1	6
8	20	8·58	0·6	1
9	20	12·4	1	3
10	20	16·25	1	3
11	20	20·45	1	3
12	20	24·6	1	5
13	20	28·2	2	6
14	10	12·55	0·6	1
15	10	16·45	0·6	1
16	10	20·65	0·6	1
17	10	24·8	1	3
18	10	29·9	1	5
19	10	34·4	1	6

The equilibrium temperature of the models was measured by the steady-state method. The model placed in the flow was kept there till its temperature stopped changing with time. Then measurements were taken. Under the experimental conditions the models can be considered "absolutely" heat-conducting ($\alpha l/k_{Cu} < 10^{-5}$). To ensure good accuracy, all the experiments on the equilibrium temperature study were carried out at stagnation temperature equal to the room temperature (the air was supplied to the nozzle from the room where the pressure chamber was installed). In this case the wall temperatures of the nozzles and of the pressure chamber which were also equal to the room temperature differed slightly from the equilibrium temperatures of the models, therefore the radiant heat fluxes to the model (or from it) were very small in all the experiments. The calculations showed that they could be neglected when determining T_e. All the estimates have shown that heat fluxes from the thermocouple head to the holder can also be neglected. The method of mean equilibrium temperature determination used in the experiments correspond to the following way of averaging the local equilibrium temperatures [2]

$$T_e = \int_0^l St_x T_{ex} x \, dx \Big/ \int_0^l St_x x \, dx. \tag{1}$$

In each run dealing with the study of the equilibrium temperature the stagnation pressure p_0 (in front of the nozzle), stagnation temperature T_0 (in front of the nozzle), Mach number M in the flow, equilibrium temperature of the model T_e were measured. From the results of the measurements r, Re, $(\sqrt{Re})/M \sqrt{C_e}$ were calculated. When calculating Re numbers, the values of μ were determined from the tables [7] and using the Sutherland formula.

The mean heat-transfer coefficients were measured by the transient method. In the experiment a cool model (at a temperature of $T_w \approx 300°K$) was rapidly immersed into the flow of the heated gas (in these runs $T_0 \approx 400$–$550°K$) in which it warmed up. The time variation of the model temperature was registered by an oscillograph supplied with high-sensitive loops (the variation of the model temperature by 10 degC caused the deviation of the light "spot" on the film by 40–60 mm). The oscillograms were decoded with the help of calibration graphs. The calibration was performed simultaneously with recoding directly onto the film of the oscillograph. The duration of the run was limited so that the model could warm up only by 15–25 degC. This eliminated noticeable radiative heat fluxes from the model and to it, since the temperature of the cooled walls of the pressure chamber and nozzle was approximately equal to the initial temperature of the model (according to the estimates the radiative heat fluxes were less than 1 per cent convective fluxes. Through the run the temperature factor $\sqrt{T_w}$ changed insufficiently (by 3–6 per cent). Under the experimental conditions the temperature field in the model can be considered uniform ($\alpha l/k_{Cu} < 10^{-5}$). If the radiative heat fluxes are negligible and the temperature field is uniform, we can write [1]

$$\alpha = -\frac{cG}{F}\frac{d}{d\tau}[\ln(T_e - T_w)]. \tag{2}$$

Here F is the area of the side cone surface; G is the weight of the model; c is the specific heat of copper; τ, time; α the mean heat-transfer coefficient. The value of the derivative $d/d\tau \times [\ln(T_e - T_w)]$ was determined graphically from the oscillograms. To determine T_e in this case, the values of p_0, T_0 and M measured in each run were used, together with the relation (6) established as the result of experimental study of the equilibrium temperature. The model weight G was determined by weighing (within the accuracy of $\pm 0{\cdot}0005$ g). The applied method of experimental determination of the mean heat-transfer coefficients corresponds to the following method of averaging

$$\alpha = \frac{2}{l^2(T_e - T_w)}\int_0^l q_x x \, dx. \tag{3}$$

The estimations showed that the errors in experimental determination of α, St, r could be ± 8–10; ± 15–20; ± 1–2 per cent, respectively.

The mean equilibrium temperature was studied within the range of $M = 2{\cdot}4$–8; $(\sqrt{Re})/M(\sqrt{C_e}) = 1{\cdot}4$–$35$. The experimental results for each θ were computed in the form of the relation $(r - r_k)/(r_m - r)$ versus $[M(\sqrt{C_e})/\sqrt{Re}]$

where

$$r_k = \sqrt{Pr}\left(\frac{T_0 - T_1}{T_0 - T}\right) + \left(\frac{T_1 - T}{T_0 - T}\right);$$

$$T_1 = \frac{T_0}{1 + [(\gamma - 1)/2]\, M_1^2} \tag{4}$$

Under such treatment the data for each θ do not show any influence of the Mach number (Fig. 2). The relation $(r - r_k)/(r_m - r)$ versus $[M(\sqrt{C_e})/\sqrt{Re}]$ is approximately linear and of the form

$$\frac{r - r_k}{r_m - r} = A\left(\frac{M\sqrt{C_e}}{\sqrt{Re}}\right). \tag{5}$$

Here $A = (6{\cdot}3 - \theta)(1 + 10\theta)$; θ is the vertex half-angle of the cone in radians. According to (5), the formula for the determination of the mean temperature recovery factors of a heat-conducting cone can be written in the form

$$r = \left(r_m + r_k \cdot \frac{1}{A}\frac{\sqrt{Re}}{M\sqrt{C_e}}\right) \Big/ \left(1 + \frac{1}{A}\frac{\sqrt{Re}}{M\sqrt{C_e}}\right). \tag{6}$$

In Fig. 3 the experimental data of the mean recovery factors are presented in the form of r versus $(\sqrt{Re})/M\sqrt{C_e}$. The values of r_m (dotted lines) and r_k (dashed lines) are shown in the same graphs. Solid lines represent relations which correspond to formula (6). In Fig. 3 the measured data of Drake and Maslach [8] for $\theta = 30°$ and $\theta = 5°$ are also plotted as shaded areas. The results of Drake and Maslach fall somewhat lower than the author's data (to a higher degree for $\theta = 5°$ and to a lower degree for $\theta = 30°$). In their measurements there were apparently some inaccuracies to judge from their results for high values of $(\sqrt{Re})/M\sqrt{C_e}$ ($\geqslant$10–15) when the flow round the cone was continual practically everywhere. They received $r_1 = 0{\cdot}8$–$0{\cdot}82$ rather than $r_1 = 0{\cdot}84$.

The mean heat-transfer coefficients were studied within the range of the numbers $M = 3{\cdot}8$–10 and $(\sqrt{Re})/M\sqrt{C_w} = 1$–$4{\cdot}5$ (for $\theta = 15°$ and 30°), $(\sqrt{Re})/M(\sqrt{C_w}) = 2$–$5{\cdot}5$ (for $\theta = 5°$

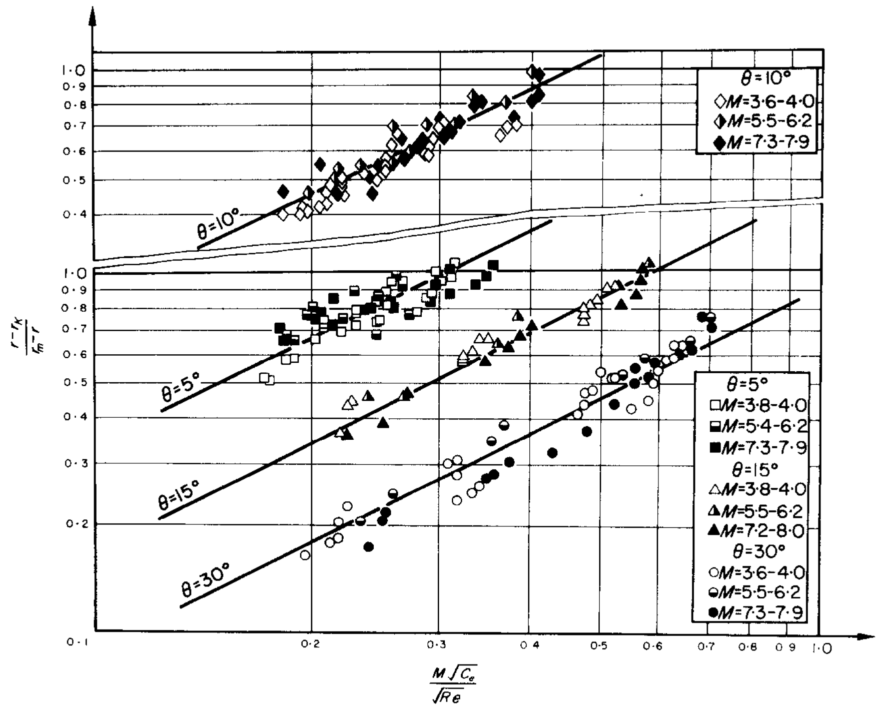

FIG. 2. Experimental relations $(r - r_k)/(r_m - r)$ vs. $[M(\sqrt{C_e}/\sqrt{Re}]$ at various θ.

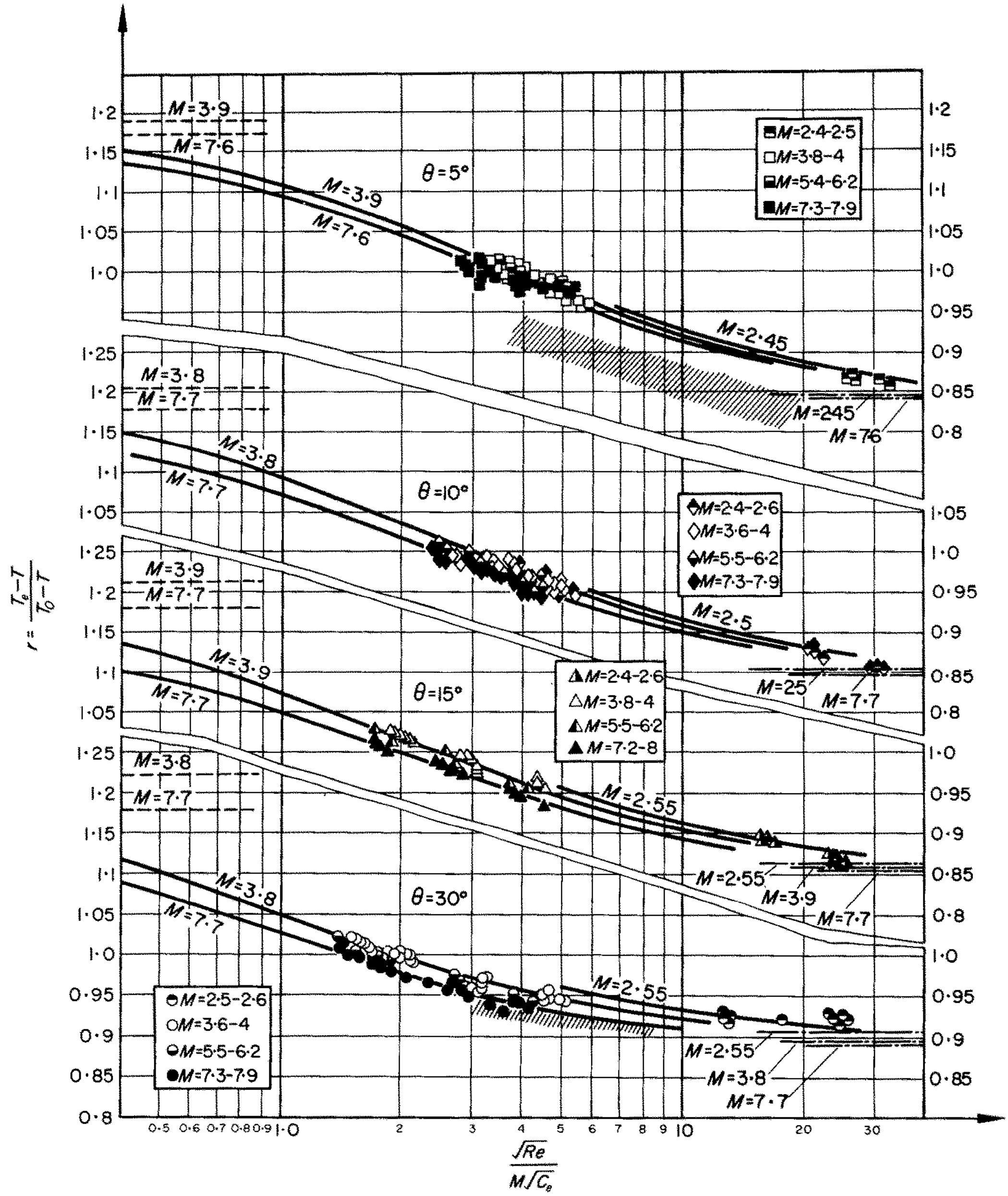

FIG. 3. Mean temperature-recovery factor of a cone in supersonic rarefied air flow. —·—·—·- continuum theory for the case without interaction between boundary layer and shock wave; — — — — free-molecular flow theory; ———— according to (6) ////////// data Drake and Maslach [8]. Author's data: □ for $\theta = 5°$; ◇ for $\theta = 10°$: △ for $\theta = 15°$; ○ for $\theta = 30°$.

and 10°). The experiments were carried out at two values of the temperature factor $\overline{T}_w = 0{\cdot}6$ (in these runs $T_0 \approx 500$–$550°K$) and $\overline{T}_w = 0{\cdot}8$ (in these runs $T_0 \approx 380$–$400°K$). The experiments covered the range of the values for the parameter of interaction η from 0·3 to 1·6.

The results of the study are plotted in Fig. 4 as St/θ^3 versus η. For comparison, the theoretical values obtained in reference [9] for the Navier–Stokes–Fourier continuum regime, accounting for the effect of curvature and the interaction between the shock wave and boundary layer, are also plotted on the same graph (solid lines). It can be seen from Fig. 4 that with the increase of the degree of rarefaction the experimental data for each θ more and more deviate from the results of theory [9] based on the Navier–Stokes–Fourier continuum equations. In this case the effect of the Mach number is not observed. The interpretation of the experimental data in the form of the relation St/St_k (here St_k is the Stanton number as obtained from theory [9]) versus the parameter $[(\sqrt{Re})/M\,(\sqrt{C_w})]$ showed that the effect of rarefaction is different depending on the value of the half-angle. The smaller θ is, the greater is this effect. This result agrees qualitatively with the theory in reference [10].

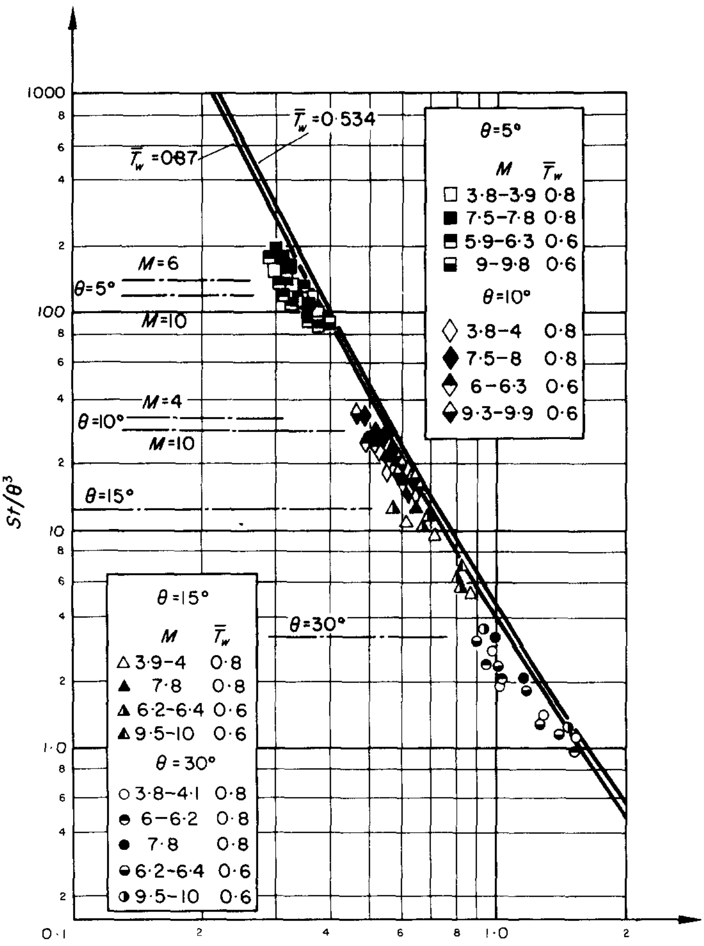

FIG. 4. Heat transfer of a cone in supersonic rarefied air flow.
——— theory [9]; —·— free-molecular flow theory [11].
Experimental data: □ for $\theta = 5°$; ◇ for $\theta = 10°$; △ for $\theta = 15°$; ○ for $\theta = 30°$.

In Fig. 4 the values of St/θ^3 are also plotted for a free molecular flow when the accomodation coefficient is 1 (dashed lines). At high M the experimental values of St numbers obtained for the cone with $\theta = 5°$ at low values of $(\sqrt{Re})/M\ (\sqrt{C_w})$ $[(\sqrt{Re})/M\ (\sqrt{C_w}) = 1–2;\ \eta = 0{\cdot}3]$ are considerably higher than the values predicted by the theory of free-molecular flow [11]. A similar though less strongly pronounced result was observed when a cone with $\theta = 10°$ was tested. With $\theta = 30°$ the experimental data at $(\sqrt{(Re)}/M\ (\sqrt{C_w}) = 1–2\ (\eta = 0{\cdot}9)$ did not practically exceed the results of the free-molecular flow theory [11]. Thus under the near free-molecular flow conditions and in the transition region heat fluxes to the cone with small θ can be essentially higher than those following from the theory of free-molecular flow. This effect decreases with the increase of θ. This experimental result confirms the theoretical considerations given in reference [12].

CONCLUSIONS

Experimental data have been obtained for heat transfer and equilibrium temperature of heat-conducting sharp cones (with vertex half-angles 5°, 10°, 15°, 30°) in a supersonic flow of rarefied gas at zero angle of attack within the range of the Mach number $M = 2{\cdot}5–10$; $Re = 25–5500$; $(\sqrt{Re})/M\ (\sqrt{C}) = 1–35$.

An empirical formula has been established for the determination of the mean recovery factors with flow regimes intermediate between that of free-molecular flow and the Navier–Stokes–Fourier continuum. It was found that under near free-molecular flow conditions, heat fluxes to the cone can be considerably higher than those predicted by the theory of free-molecular flow. Slip effects depend on the vertex angle.

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Abstract—The methods and results are reported of experimental studies of the equilibrium temperature and heat transfer of sharp cones (with vertex half-angles of 5°, 10°, 15°, 30°) in a rarefied air flow at zero angle of attack and $M = 2{\cdot}5–10$; $Re = 25–5500$; $(\sqrt{Re})/M(\sqrt{C}) = 1–35$.

Zusammenfassung—Es wird über Methoden und Ergebnisse einer experimentellen Untersuchung berichtet über die Gleichgewichtstemperatur und den Wärmeübergang an spitzen Kegeln (mit Scheitelhalbwinkeln von 5°, 10°, 15°, 30°) in einem verdünnten Luftstrom und Anströmwinkel von 0° und $M = 2{,}5–10$; $Re = 25–5500$; $(\sqrt{Re})/M(\sqrt{C}) = 1–35$.